

## ■ ORIGINAL PAPER ■

# Flame Visualization and Mechanism of Fast Flame Propagation through a Meso-scale Packed Porous Bed in a High-pressure Environment

OKUYAMA, Masaki<sup>1\*</sup>, SUZUKI, Takuro<sup>2</sup>, WANG, Jinhuwa<sup>3</sup>, and KOBAYASHI, Hideaki<sup>2</sup><sup>1</sup> Yanmar Corporation R&D Center, Maibara, Shiga 521-8511, Japan<sup>2</sup> Institute of Fluid Science, Tohoku University, Sendai, Miyagi 980-8577, Japan<sup>3</sup> State Key Laboratory of Multiphase Flow in Power Engineering, Xi'an Jiaotong University, Xi'an, 710049, P.R. China

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**Abstract :** To understand the high-pressure premixed combustion mechanism in a meso-scale packed porous bed, cross-sectional visualization of propagating flames in a packed bed was performed at high pressure up to 1.0 MPa. For easy optical access to the interior, a two-dimensional pseudo packed bed combustor was developed, which has quartz-glass cylinders installed in a rectangular duct representing the network of meso-scale flow channels of the bed. Visualization of propagating flames in the 2-D packed bed was performed using both a visible-light high-speed photography and Planar Laser Induced Fluorescence (PLIF) method targeting OH radicals. Fast flame propagation through the 2-D packed bed with a broad chemiluminescence region was observed at high pressure, characteristics of which was presumed to be identical to the fast flame propagation in the packed bed of spheres in authors' previous research. Instantaneous cross-sectional visualization with OH-PLIF was subsequently conducted. Obtained PLIF images as well as the result of turbulence measurement in the 2-D packed bed showed that turbulent flames with concave and convex cusps can be formed even in meso-scale flow channels at high pressure, resulting in a high flame displacement speed. The mechanism of the turbulent flame formation in quite a narrow flow channel was subsequently explained by comparison among characteristic scales of turbulence, hydrodynamic flame instability combined with diffusive-thermal effect, and flow channels. OH-PLIF images also revealed the existence of multiple propagating flame fronts inside the broad chemiluminescence region, in which the leading flame front was presumed to be predominant on the flame propagation in the packed bed.

**Key Words :** Turbulent flame propagation, Porous media, Packed bed, High-pressure environment, Planar Laser Induced Fluorescence

## 1. Introduction

Owing to its unique characteristics and advantages, a packed pebble bed as typical porous media is utilized as an element of various combustors and reactors, e.g., ultra-lean burners, gas infrared burners, fuel-reforming reactors and also flame arrestors [1, 2]. For sophisticated design and safe operation of such devices, understanding the premixed combustion characteristics in a packed pebble bed is essential. Especially for industrial purpose, it is important to clarify the flame propagation mechanism in a packed bed under a high-pressure environment where the practical combustor and reactor are operated. Although most preceding studies on combustion in a packed bed are limited to the atmospheric pressure condition, several experimental studies have revealed the unique characteristics of

high-pressure premixed combustion inside a packed bed.

Korzhavin et al. [3] and Babkin et al. [4], for instance, experimentally investigated the premixed flame propagation in the packed bed of spheres with diameters of about 5-10 mm installed in the closed vessel at high pressure up to 1.5 MPa. They reported that the thick flame region with widely distributed chemiluminescence (typically 3-5 cm in height) propagated through the bed. Measured flame displacement speed at high pressure was more than ten times faster than the laminar flame speed, and it also increased with initial pressure in the vessel. As for the fast flame propagation at high initial pressure, they pointed out in the article that the flame speed was accelerated by turbulence.

Recently, author's research group [5] conducted the flame propagation experiment in a packed pebble bed over a wide range of ambient pressures and convective gas flow velocities through the bed. Broad chemiluminescence region rapidly

\* Corresponding author. E-mail: masaki\_okuyama@yanmar.com

propagating in the bed was also observed as was in the above-mentioned works [3, 4]. It was interesting to find that the flame speed normalized by the laminar burning velocity of the mixture had the minimum value around the critical Reynolds number, and then it sharply increases with increase in both gas flow velocity and ambient pressure. Authors also performed the turbulence measurement inside a flow channel of a pseudo two-dimensional packed bed, and clarified that the flow inside a packed bed transits from laminar to turbulent over a critical Reynolds number mentioned above [5]. These experimental results indicate that the premixed combustion mechanism in a packed bed at sufficiently high pressure is dominated by that of turbulent premixed combustion.

However, it is not yet clearly proven that the turbulent premixed flames with a thin wrinkled flame fronts interacted with turbulence can be formed in quite a narrow flow channel of the packed bed, a breadth of which is typically several millimeters, i.e., meso-scale. Here, combustion in meso-scale space is defined as the phenomena occurring in the intermediate region between the micro space whose scale is close to or smaller than the quenching distance and the open macro space. Besides, the internal structure of broad chemiluminescence region observed in the previous study [3, 5] and the way that the flame propagates through a network of flow channels in the bed are also of interest. In this context, for the further understanding of the high-pressure premixed combustion characteristics in a packed bed, instantaneous direct visualization of propagating flames inside the flow channel of a packed bed is desired.

The purpose of the present study is, therefore, to explore the premixed combustion mechanism in a packed bed at high pressure by visualizing the instantaneous flame structures in the bed. Especially, the local flame front structure in the broad flame region was focused on to examine the assumption of turbulent premixed combustion in a meso-scale flow channel network of the bed. Since the direct observation and visualization of flames inside a packed pebble bed are quite difficult due to the limited optical access to the interior, an attempt is made to use a newly developed two-dimensional (2-D) pseudo packed bed combustor with easy optical access to its interior. Firstly, flame behavior in a 2-D packed bed combustor was observed using a high-speed photography, and it was compared to that in a packed bed of spheres. Subsequently, cross-sectional flame visualization using the Planar Laser Induced Fluorescence (PLIF) method targeting OH radicals was attempted.

## 2. Experimental Setup and Procedure

Figure 1 shows front, top views and a schematic of a flow channel network of the two-dimensional (2-D) pseudo packed bed combustor developed in the present study. Hollow tubes

with an outer diameter of 10 mm were installed in a staggered alignment inside a rectangular duct (cross section of 61.2 x 16.5 mm), which represent the network of meso-scale flow channels of the packed pebble bed. In the present study, hollow quartz-glass tubes with thickness of 1 mm were used so that the ultraviolet laser sheet could be introduced to the flow channel among glass tubes as an inspection area. Distances between tubes were set at 2.7 mm and 1.0 mm as shown in Fig. 1, which are derived from the hydraulic diameter of void planes surrounded by four and three pebbles with a diameter of 10 mm. The flow channel structure of the present combustor is exactly same as the 2-D packed bed used for turbulence measurement in the author's previous study [5]. Since flow field characteristics in the combustor, i.e., mean gas flow velocity, turbulence intensity profiles and turbulence scales, were measured with the hotwire anemometry (Dantec Dynamics, Streamline 90N10) in the previous work [5], those data are used again for discussion in the present study.

For direct flame observations and photography, 2-D packed bed combustor has a quartz-glass front panel. It also has quartz-glass side windows for the laser sheet introduction in cases of OH-PLIF measurements. A laser sheet with the breadth of 50 mm and the width of less than 100  $\mu\text{m}$  was formed using a beam expander, and then introduced perpendicular to the bundle of quartz tubes through a narrow rectangular slit on both sides of the burner. Since the thickness of the quartz tube is sufficiently small, it is presumed that the laser power profile in each flow channel is basically uniform. The introduced laser sheet was

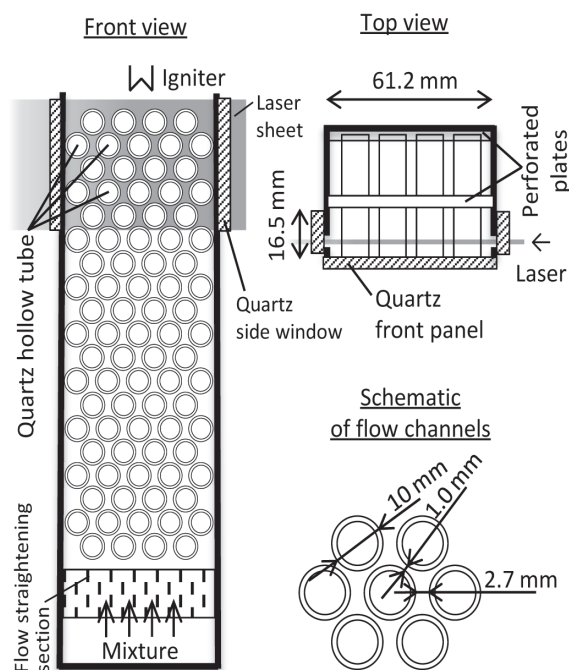


Fig. 1. Front and top views of the 2-D pseudo packed bed combustor for flame visualization and the schematic of flow channel network in the bed.

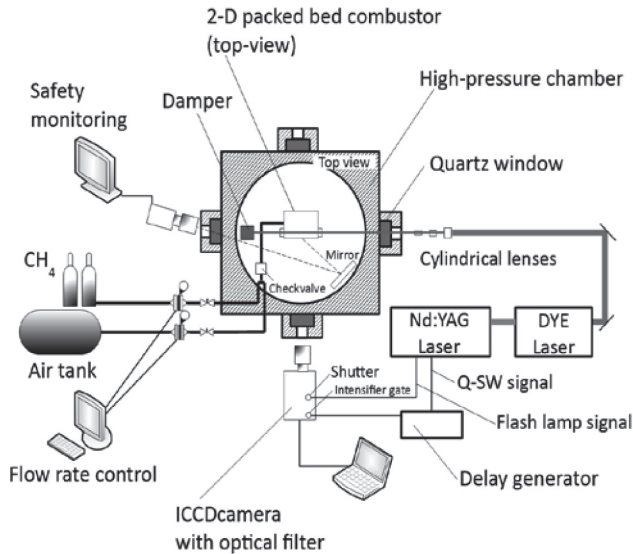


Fig.2. Top view of the experimental setup for high-pressure combustion test and OH-PLIF flame visualization.

5 mm away from the front quartz panel, where the effect of wall boundary layer can be neglected according to the result of preliminary conducted velocity profile measurement using the hotwire anemometry. The laser sheet coming out of the burner was finally captured by a damper.

Figure 2 shows a top-view schematic of experimental setup for high-pressure combustion test. The 2-D packed bed was installed in the high-pressure combustion test facility of IFS, Tohoku University [6] to keep the ambient pressure and temperature constant during the experiment under the existence of convective gas flows through the combustor. The fuel-lean CH<sub>4</sub>/air mixture at room-temperature was supplied to the burner from its bottom by way of pre-calibrated orifice flow meters. The mixture was introduced to the layer of quartz-glass cylinders through a ceramic honeycomb and a perforated plate to make the velocity profile uniform. It was then ignited at the exit of the burner with the electrically heated Nichrome-wire.

Flame behavior in the 2-D packed bed was observed by using both a visible-light high-speed video camera (Casio EX-F1, frame rate of 300 fps and exposure time of 1/320 s) and the OH-PLIF method, detail of which is noted here. As a laser source, a tunable DYE laser (Lumonics, HD-500) with a frequency doubler (Lumonics, HT-1000) was used. As a pump source, second harmonics at 532 nm of the Nd:YAG Laser (Spectra Physics, GCR-250) was used. The maximum energy of single shot laser was about 15-17 mJ. A Q1(6) branch of the OH(1,0) band, i.e., wavelength of 282.928 nm, was chosen for the excitation, and almost all fluorescence from OH(0,0) band was detected using the intensified CCD camera with a full resolution of 1024 x 1024 pixels (Andor Technology, iSTAR DH-334T) and also a UV lens (Nikon, UV-105 mm F4.5S). The

exposure time of CCD and a gate width of the image intensifier were set at 11.0 ms and 50.0 ns, respectively. The spatial resolution of attained images was approximately 50  $\mu\text{m}/\text{pixel}$ , which was sufficiently small to capture small flame wrinkles of high-pressure turbulent flames [6]. Since a strong reflection of a laser light at the surface and inside of quartz tubes appeared as a major part of the measurement noise, the interference-type UV band pass filter (Asahi Spectra, LX0313, central wavelength = 314.5 nm, FWHM = 11.0 nm) with the broad band pass filter (Schott, UG11) was employed to distinguish the PLIF signals from such reflection noise.

### 3. Results and Discussion

#### 3.1. Observation and Flame Behaviors in a 2-D Packed Bed

Prior to OH-PLIF measurements, observation of the flame behavior in the 2-D packed bed at various ambient pressures was performed using the visible-light high-speed video camera. Figure 3 shows the representative flame images in the 2-D packed bed combustor at pressures of  $P = 0.1$  MPa and 1.0 MPa. At atmospheric pressure, only the flame stabilized at the burner exit was observed, not the propagating flame even in a case of quite a low gas flow velocity,  $U$ , of 5 and 10 cm/s, as shown in Fig. 3(a). Here,  $U$  is defined as the superficial velocity, i.e., the volumetric flow rate divided by the cross-sectional area without tubes. When  $U$  increased up to 60 and 90 cm/s, transition from a laminar Bunsen-type flame to an unsteady turbulent flame with a thick flame brush was observed. Pictures in Fig. 3(a) are taken at approx. 20 seconds after the ignition, and the flame in the figure can be considered as the result under the steady-state condition.

Whereas, at high pressure, flame propagation through the bed in the upstream direction was observed in case of a relatively low gas velocity, as well as the stabilized flame in the first bank of cylinders in case of a sufficiently high gas velocity. Figure 3(b) shows the typical serial images of the flame propagation through bed at the pressure,  $P$ , of 1.0 MPa and  $U$  of 10 cm/s. Flames were firstly stabilized in the first bank of cylinders for a while, and then propagated upstream with widely distributed chemiluminescence, which should be treated as a flame region. Here, an effect of the exposure time on the flame region thickness is negligible since the travel distance of the flame region forehead within an exposure time was less than 2 mm in height. The calculated displacement speed of the flame region forehead while constant was approximately 46 cm/s, which was more than three times faster than the laminar burning velocity of the mixture, 13.4 cm/s, calculated using PREMIX [7] with CHEMKIN-II database [8] and GRI-Mech ver3.0 [9]. It can be said that this fast flame propagation with broad flame region observed in the 2-D packed bed combustor should be identical

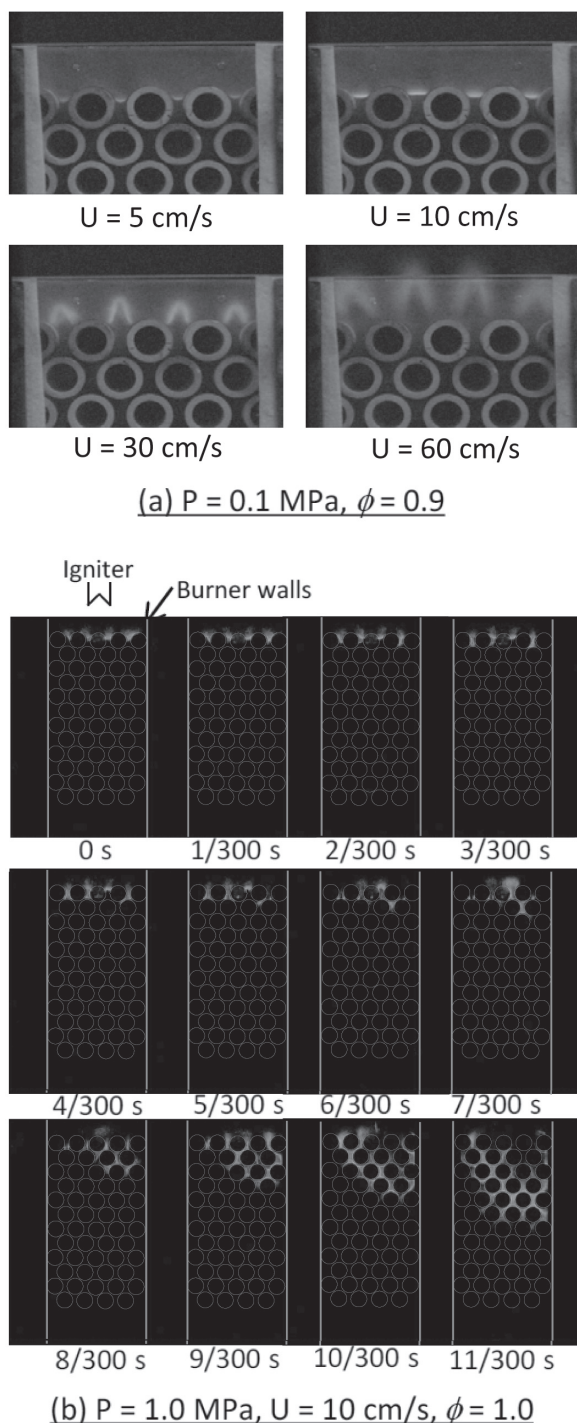


Fig.3. Representative  $\text{CH}_4/\text{air}$  flame behaviors in a 2-D packed bed at atmospheric and high pressure of 1.0 MPa.

to that in the packed bed of spheres observed by Korzhavin et al. [3] and authors [5], indicating the validity of the 2-D packed bed modeling of this study. Here again, the obtained flame image was sum of the chemiluminescence in a depth direction, to further analyze the actual flame structure inside the broad chemiluminescence zone, instantaneous cross-sectional visualization of propagating flames is necessary. Therefore, in the present study, OH-PLIF visualization for propagating and

also stabilized flames in the 2-D packed bed was subsequently attempted.

### 3.2. Local Flame Front Structure Interacted with Flow Turbulence in a Packed Bed at High Pressure

Figure 4 shows representative OH-PLIF images of propagating and stabilized flames in the 2-D packed bed with various ambient pressures,  $P$ , of 0.1 to 1.0 MPa, and flow velocities,  $U$ , of 30 to 100 cm/s. At atmospheric pressure, as mentioned above, only the flames stabilized outside of the burner were observed. Flame fronts were laminar at  $U = 30 \text{ cm/s}$ , and then became turbulent at  $U$  larger 60 cm/s. At atmospheric pressure, flame could not propagate into the flow channel of the packed bed. On the other hand, at high pressure of 0.5 and 1.0 MPa, OH-PLIF images of propagating flames were successfully obtained as well as those of stabilized flames at the first bank of quartz cylinders. In case of propagating flames, at best single OH-PLIF image could be obtained for single flame propagation in the present experiment since flame displacement speed was quite fast. Time for the flame region to pass through the camera view was less than a shot interval of Nd:YAG laser, i.e., 100 ms.

When we focus on the flame front structure of propagating flames at high pressure of 0.5 MPa and 1.0 MPa, it is important to see that the flame with small wrinkles was formed in single flow channel surrounded by quartz-glass tubes after the flame entered into the bed, e.g.,  $U = 30 \text{ cm/s}$  at  $P = 0.5 \text{ MPa}$  and  $U = 40 \text{ cm/s}$  at  $P = 1.0 \text{ MPa}$ . Here, for further discussion, result of flow turbulence measurement in the 2-D packed bed reported in authors' previous research [5] is again presented in Fig. 5. The figure shows the relative turbulence intensity, defined by turbulence intensity,  $u'$ , divided by local mean flow velocity,  $U_{\text{local}}$ , at the center of each pore along the center axis of 2-D packed bed measured by hot-wire anemometry. From Fig. 5, it was shown that flow field in the 2-D packed bed is fully developed turbulent at high pressure of 0.5 MPa and 1.0 MPa for all conditions shown in Fig. 4. It is known that flow inside the packed pebble bed becomes turbulent when Reynolds number based on the pebble diameter reaches around several hundred, of which Reynolds number is quite smaller compared to the case of ordinal pipe flows [5]. Accordingly, it is reasonable to consider that flame front structure with small wrinkle and corrugation presented in Fig. 4 is caused by gas flow turbulence interacted with flame.

Then, it is interesting to see that flame front had one slightly curved cusp in a single flow channel at  $P = 0.5 \text{ MPa}$  and  $U = 30 \text{ cm/s}$ ; on the other hand, at  $P = 1.0 \text{ MPa}$  and  $U = 40 \text{ cm/s}$ , three cusps in a single pore were observed. This indicates that the flame front structure became finer with pressure increase as was observed by Kobayashi et al. [10] in the case of open Bunsen flames stabilized at the nozzle outlet. It is also interesting to see



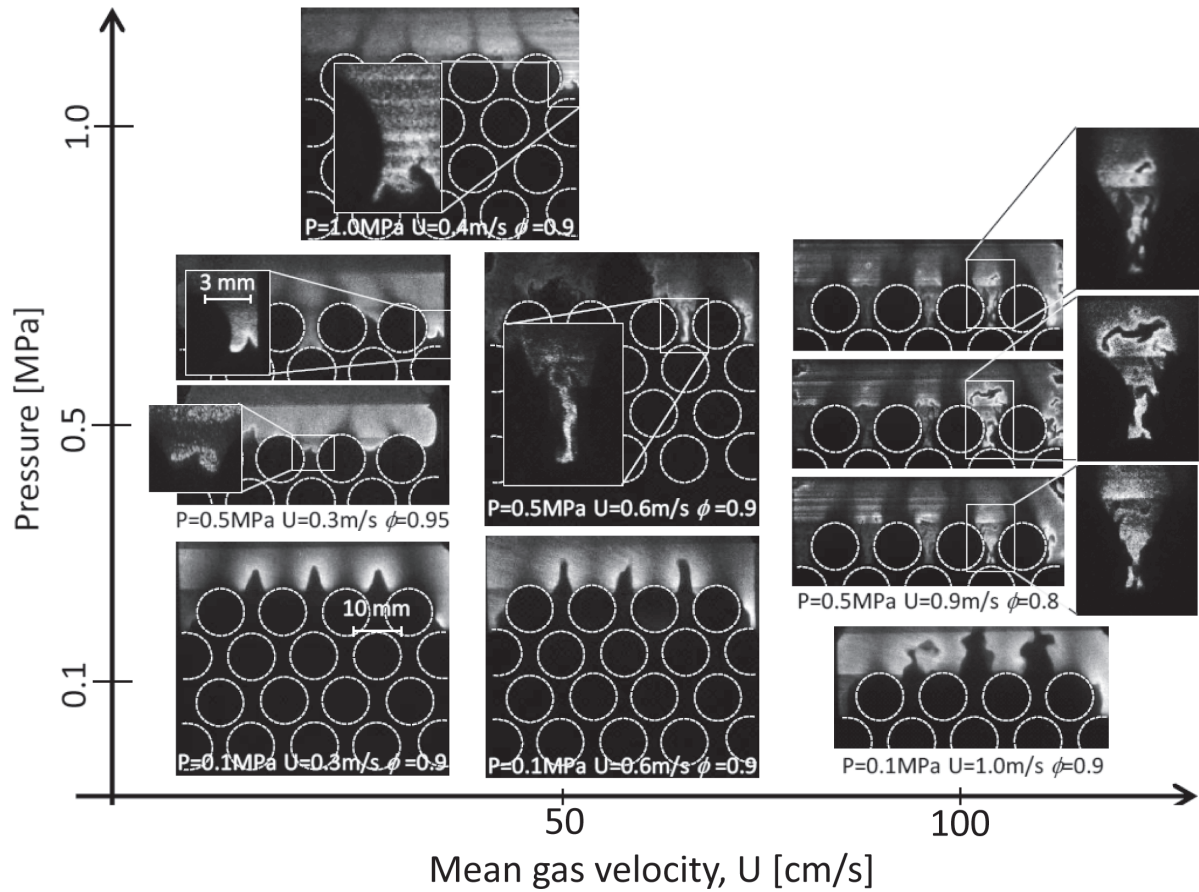


Fig.4. Representative OH-PLIF images of CH<sub>4</sub>/air propagating and stabilized flames in a 2-D packed bed at various pressures,  $P = 0.1$  to  $1.0$  MPa, convective gas velocities,  $U = 30$  to  $100$  cm/s, and equivalence ratios,  $\phi = 0.8$ - $0.95$ .

that flame front structure became more complex and convoluted with increase in gas flow velocity at  $P = 0.5$  MPa and  $U = 30, 60, 90$  cm/s. This is presumed to be caused by increase in turbulence intensity according to the turbulence measurement result. Based on these discussion, although number of obtained PLIF images is limited due to experimental difficulty, one can conclude that the observed flame structures of propagating flames in Fig. 4 as well as turbulence measurement results in Fig. 5 supports the discussion in authors' previous study [5] that the turbulent flame can be formed even in the meso-scale flow channels in a packed pebble bed at sufficiently high pressure. As a result, the increase in flame speed with increase in pressure in porous media reported in several works [3-5] can be explained by total flame area increase with finer flame cusps at elevated pressure.

Here, the reason why turbulent flame formation is possible even in meso-scale flow channels in the packed bed at high pressure is examined by a comparison of characteristic length scales of the flame, turbulence, boundary layer, and the flow channel. According to the explanation by Kobayashi et al. [10], the mechanism of finer flame front structures at high pressure is due to both the decrease in an average vortex tube diameter which is close to 10 times the Kolmogorov scale,  $10 \eta_k$  [11], and

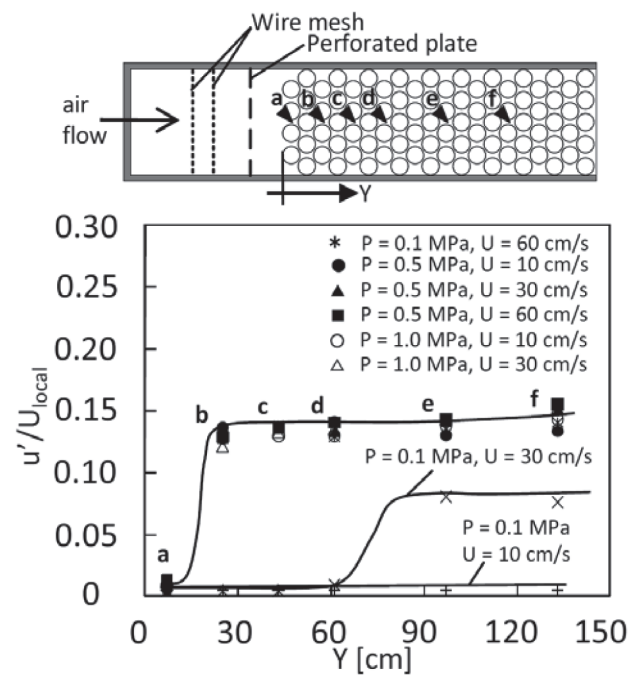


Fig.5. Measured relative turbulence intensities profile along the center axis of the 2-D pseudo packed bed [5].

the decrease in the characteristic scale of hydrodynamic flame instability combined with diffusive-thermal effect,  $l_i$ , evaluated using Sivashinsky's formulation [12] which corresponds to a wavelength at the maximum growth rate of flame front disturbance [10]. Besides, in case of premixed combustion in a packed bed, a boundary layer thickness along the solid wall,  $d_b$ , and a quenching distance,  $d_q$  ( $\sim 40 \delta$  [13], where  $\delta$  is laminar flame thickness), should be additionally considered. The boundary layer thickness was evaluated as that of a turbulent boundary layer which develops at a trailing edge of the flat plate, i.e.,  $d_b = 0.37x(U_\infty \nu)^{-1/5}$  ( $U_\infty$ : free stream velocity outside the boundary layer,  $\nu$ : kinematic viscosity  $x$ : length of the plate [14]). Here, the free stream velocity and the plate length were set at 1.0 m/s and 10 mm, respectively. The quenching distance and the characteristic scale of flame instability were evaluated for CH<sub>4</sub>/air mixture at equivalence ratios of 0.8, 0.9 and 1.0. Kolmogorov scales were derived from results of turbulence measurements with a hot-wire anemometry in authors' previous research [5] with an inlet gas velocity of 30 cm/s. It is presumed here that the effect of equivalence ratios on a boundary layer thickness and an average vortex diameter is small since the variation of kinematic viscosity for these mixtures is negligible.

Figure 6 shows the variation of characteristic length scales, i.e.,  $d_q$ ,  $d_b$ ,  $10\eta_k$ , and  $l_i$ , with increase in pressure up to 3.0 MPa. It is clear that these all characteristic length scales become smaller with pressure rise. Decrease in the boundary layer thickness and the quenching distance means that effects

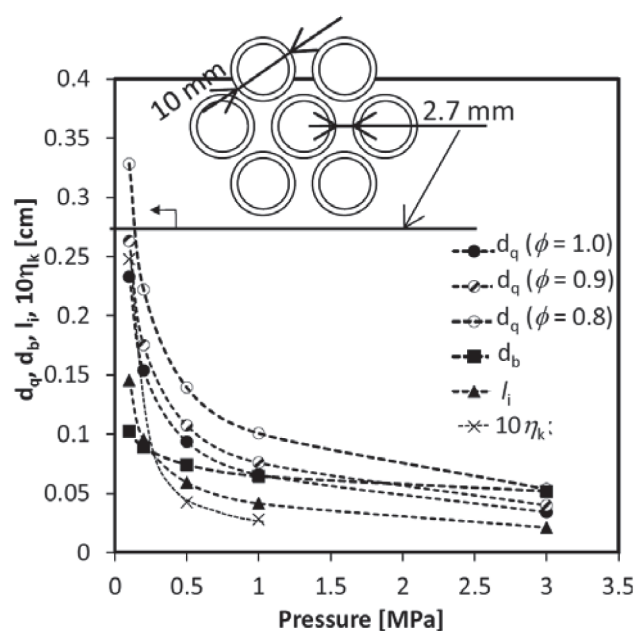


Fig.6. Variations of a quenching distance,  $d_q$ , an average diameter of vortex tubes,  $10\eta_k$ , a turbulent boundary layer thickness,  $d_b$ , and a characteristic scale of flame instability,  $l_i$ , with ambient pressures. In the figure, a breadth of flow channels in a 2-D packed bed is also shown for comparison.

of the solid wall on the flow field and flames are limited at high pressure. Decrease in the average vortex tube diameter in turbulent flows, and characteristic scale of flame instability should be main reason why turbulent premixed flames can be formed even in meso-scale flow channels at high pressure. Actually,  $10\eta_k$  and  $l_i$  are less than 1/5 of the flow channel breadth at  $P = 1.0$  MPa. On the other hand, at atmospheric pressure, the average vortex tube diameter is almost the same as the flow channel breadth, which means that turbulent flame front cannot be formed in the flow channel of this study. Considering the fact that the turbulent flame with several concave and convex cusps were seen in a single pore at high pressure, fundamental combustion characteristics in the packed bed, i.e., flame speed, flame stability and flammability limit, are presumed to be affected by turbulence-flame interaction at high pressure.

### 3.3. Flame Propagation Pathways and an Initiation of Flashback in a Packed Bed

Finally, the reason why the broad flame region is formed and propagates in the packed bed is discussed here by visualizing flame propagation pathways in the flow channel network with OH-PLIF. Figure 7 shows two representative OH-PLIF images of flames propagating deeply into fourth or fifth bank of cylinders at  $P = 0.5$  MPa and  $U = 30$  cm/s. Widely distributed OH-PLIF signal was seen in both images. Considering the PLIF signal intensity profile shown in the inset of Fig. 7, upstream boundaries of OH-PLIF signal correspond to the propagating flame fronts. From Fig. 7, it was seen that the flame propagation upstream and downstream happened simultaneously in the 2-D cross-section. That is, several flame fronts simultaneously existed in the flow channel network, one of which is a leading edge of the flame region and others are followers. The last flame front were delayed by 3 or 4 banks of cylinders compared to the most advanced flame front in the packed bed, i.e., 3-4 cm in height, which was in reasonable agreement with the observation of high-speed video images shown in Fig. 3(b). It is natural that the unsteady behavior of turbulent flame fronts results in an apparent thick flame region which is so called flame brush; however, it is not enough to explain the observed broad flame region up to 3-4 cm in the packed bed. Thus, it is reasonable to consider that these multiple flame fronts simultaneously existing in several flow channels in the packed bed are major reason for the broad flame region and overall fast flame propagation.

It can be also said from Fig. 7 that the leading flame front plays an important role in initiating and sustaining the flame propagation in the bed. This is because the flame propagation in the downstream direction is straightforward, meaning that a single flame front propagation in a single flow channel is thought to drive the flashback in the whole system. It was also indicated that the flame displacement speed in the packed bed is dominated

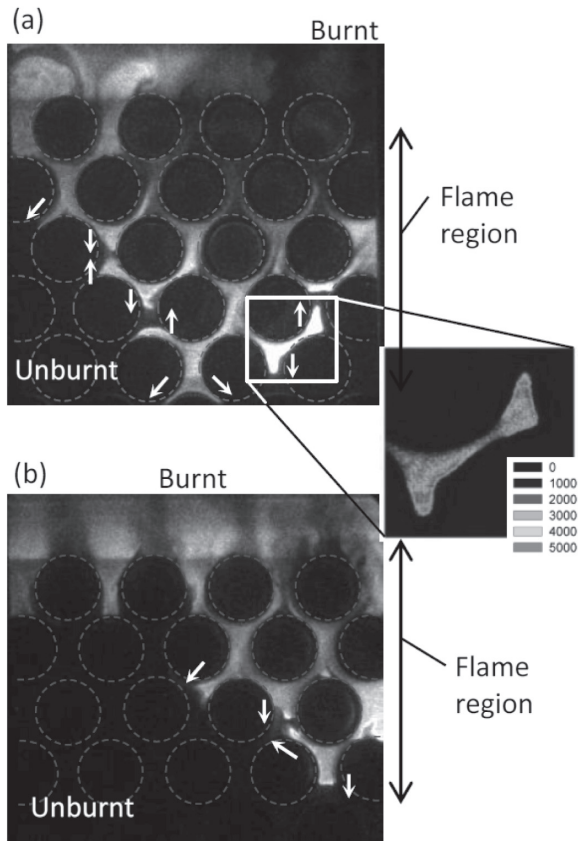


Fig.7. Visualized flame propagation pathways in a flow channel network of a 2-D packed bed at high pressure ((a)(b)  $P = 0.5$  MPa,  $U = 30$  cm/s,  $\phi = 0.95$ ). Inset shows LIF signal intensity profile. Arrows in the figure denotes the direction of the flame front propagation.

by that of the leading flame front interacted with turbulence. These perceptions are important when the packed pebble bed is utilized as a flame arrestor.

#### 4. Conclusions

An attempt was made to visualize the propagating flames through the meso-scale flow channel network of a packed bed to examine the high-pressure premixed flame propagation mechanism in a packed porous bed. The following conclusions are obtained:

1. At high pressure, flames with widely distributed chemiluminescence region propagated through the 2-D packed bed. Measured flame displacement speed was quite higher than the laminar burning velocity. These characteristics are said to be identical to those of fast flame propagation in the packed bed of spheres at high pressure observed in the previous work by Korzhavin et al. and the authors of the present study.
2. The flame front structure visualized with OH-PLIF as well

as the turbulence measurement results in the 2-D packed bed indicated that turbulent premixed combustion can be realized inside the meso-scale flow channels of the bed at high pressure, resulting in a high flame displacement speed. This is presumed to be caused by the decrease in average vortex tube diameter and characteristic scale of flame instability with pressure increase.

3. OH-PLIF images of the propagating flame showed that several flame fronts simultaneously exist inside the 2-D cross section of the broad flame region, some of which propagated upstream and others downstream. This multiple flame fronts in the bed can explain the mechanism of overall fast flame propagation accompanied by the broad chemiluminescence region.

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